Generalized Random Sign and Alert Delay Models for Imperfect Maintenance

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Context

Context

All along their life, complex industrial systems are subjected to two kinds of maintenance:

- **Corrective Maintenance (CM, repair)**:
  after a failure (burn-in defects, wear-out), and intends to put the system functional again.
- **Preventive Maintenance (PM)**:
  while the system is in operational conditions, and intends to slow down the wear process and reduce the frequency of occurrence of failures. Condition based PM are carried out according to the results of inspections and degradation or operation controls.

$\rightarrow$ To present a modelling of the dependency between corrective and condition-based maintenances considering imperfect maintenance efficiency.

$\rightarrow$ Extend competing risks models initially defined for perfect maintenance.

Modelling the maintenance process

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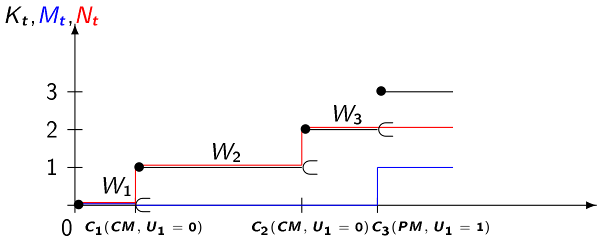

- Times of maintenance (PM and CM): $\{C_i\}_{i \geq 1}$
- Inter-maintenance times (PM and CM): $W_i = C_i - C_{i-1}, \; i \geq 1$
- Types of maintenance:

$$U_i = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ maintenance is preventive} \\ 0 & \text{otherwise} \end{cases}$$

- Counting processes: $\begin{cases} \{K_t\}_{t \geq 0} & \text{PM and CM} \\ \{N_t\}_{t \geq 0} & \text{CM} \\ \{M_t\}_{t \geq 0} & \text{PM} \end{cases}$

Stochastic modelling

The maintenance intensities:

- The global maintenance intensity:
$\lambda_t^K(K,U) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(K_{t+\Delta t} - K_{t^-} = 1 | \mathcal{F}_{t^-})$
- The corrective maintenance intensity:
$\lambda_t^N(K,U) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(N_{t+\Delta t} - N_{t^-} = 1 | \mathcal{F}_{t^-})$
- The preventive maintenance intensity:
$\lambda_t^M(K,U) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(M_{t+\Delta t} - M_{t^-} = 1 | \mathcal{F}_{t^-})$

where $\mathcal{F}_t$ corresponds to the history of the process at t.

Usually, $\mathcal{F}_t = \sigma\left(\{K_s, U_{K_s}\}_{0 \le s \le t}\right)$.

- $\lambda_t^K(K,U) = \lambda_t^N(K,U) + \lambda_t^M(K,U)$

$\rightarrow$ PM and CM intensities can entirely define the maintenance process as the likelihood function for estimation purposes.

The competing risks framework

The competing risks framework

Cooke and Paulsen (1994)

After the k^{th} maintenance, we define two **risk variables** :

- Y_{k+1} = potential time to next maintenance, if it is a PM (PM risk).
- Z_{k+1} = potential time to next maintenance, if it is a CM (CM risk).

Observations

In practice, Y_{k+1} and Z_{k+1} are not observed. The actual observations are:

- The inter-maintenance time: $W_{k+1} = \min(Y_{k+1}, Z_{k+1})$
- The next kind of maintenance :

$$U_{k+1} = \begin{cases} 1 & \text{if} \quad Y_{k+1} < Z_{k+1} \quad \text{(PM)} \\ 0 & \text{if} \quad Z_{k+1} < Y_{k+1} \quad \text{(CM)} \end{cases}$$

Definitions and notations

Joint survival function of (Y_1, Z_1)

$$S(y,z) = P(Y_1 > y, Z_1 > z)$$

Sub-survival functions

$$S_Z^*(z) = P(Z_1 > z, Z_1 < Y_1) = P(W_1 > z, U_1 = 0)$$

$$S_Y^*(y) = P(Y_1 > y, Y_1 \leq Z_1) = P(W_1 > y, U_1 = 1)$$

Distributions of W_1 and U_1

$$S_{W_1}(w) = S_Y^*(w) + S_Z^*(w)$$

$$P(U_1 = 1) = P(Y_1 \leq Z_1) = S_Y^*(0)$$

The diagnostic function ϕ: Probability of PM beyond w

$$\phi(w) = P(Y_1 \leq Z_1 | W_1 > w) = P(U_1 = 1 | W_1 > w) = \frac{S_Y^*(w)}{S_Y^*(w) + S_Z^*(w)}$$

Usual competing risks models (UCR)

Independent risks model (IUCR)

$$Y_1 \perp Z_1$$

$\rightarrow$ Easy computations but not a realistic assumption considering PM and CM.

Proportional Hazards model (PH)

$$W_1 \perp U_1$$

$\rightarrow$ ϕ is constant.

Delay Time model (Christer [02])

$$Y_1 = A + C, \qquad Z_1 = B + C$$

where A, B and C are mutually independent random variables.

$\rightarrow$ When A and B are exponentially distributed, it is a PH model and ϕ is constant.

The Random Sign Assumption (Cooke [93])

Definition: $U_1 \perp Z_1$

$\longrightarrow \phi$ is maximum at the origin.

The Repair Alert (RA) model (Lindqvist-Stove-Langseth [06]):

- Random sign assumption
- $P(Y_1 \leq y | Z_1 = z, Y_1 < Z_1) = \dfrac{G(y)}{G(z)}, \qquad G(0) = 0, G$ is increasing.
- $q = P(Y_1 < Z_1) = P(U_1 = 1)$

Intensity Proportional Repair Alert (IPRA) model: $G = \Lambda_{Z_1}$: ϕ is decreasing.

$\rightarrow$ Other models based on the Random Sign assumption such as the Highly Correlated Censoring model (Bunea and Bedford [02])

The alert-delay model (AD - Dijoux, Gaudoin [09])

Definition

$$Y_1 = pZ_1 + \mathcal{E}$$

- $p \in [0, 1]$.
- $Z_1 \perp \mathcal{E}$.
- Z_1 and $\mathcal{E}$ positive random variables.

The exponential alert-delay model

- $Z_1 \sim Exp(\lambda)$
- $\mathcal{E} \sim Exp(\mu)$

$\rightarrow$ ϕ is increasing.

The generalized competing risks models

Principle: To generalize the UCR approach by using the past of the maintenance process in order to take into account imperfect maintenances: the $\{(\mathbf{W}_i, \mathbf{U}_i)\}_{1 \leq i \leq k}$ are not iid.

Generalized conditional survival functions:

$$S_{k+1}(y, z; \mathbf{W}_k, \mathbf{U}_k) = P(Y_{k+1} > y, Z_{k+1} > z | \mathbf{W}_k, \mathbf{U}_k)$$

→ Generalization of the S^*, CS^* and ϕ functions by conditioning to the past $(\mathbf{W}_k, \mathbf{U}_k)$.

$$\phi_{k+1}(w; \mathbf{W}_k, \mathbf{U}_k) = P(U_{k+1} = 1 | W_{k+1} > w; \mathbf{W}_k, \mathbf{U}_k)$$

→ Intensities, distributions of the observations, likelihood function can be easily derived from the S_{k+1}.

$$\lambda_t^N(K, U) = \frac{\left[-\frac{\partial}{\partial z} S_{K_{t-}+1}(y, z; W_1, \ldots, U_{K_{t-}}) \right]_{(t - C_{K_{t-}}, t - C_{K_{t-}})}}{S_{K_{t-}+1}(t - C_{K_{t-}}, t - C_{K_{t-}}; W_1, \ldots, U_{K_{t-}})}$$

The generalized virtual age models (GVA)

Principle: After the k^{th} maintenance, the system behaves as a new one having been operational an age A_k without being maintained.

$$P(W_{k+1} > w, U_{k+1} = u | \mathbf{W}_k, \mathbf{U}_k) = P(W > w + A_k, U = u | W > A_k)$$

where (W, U) has the same distribution as (W_1, U_1).

$$\lambda_t^N(K, U) = \lambda_c(t - C_{K_{t^-}} + A_{K_{t^-}}), \text{ where } \lambda_c(t) = \frac{\left[-\frac{\partial}{\partial z} S_1(y, z)\right]_{(t,t)}}{S_1(t, t)}$$

Example: PM and CM are ARA_∞:

$$A_k = \begin{cases} 0 & \text{if } k = 0 \\ (1 - \rho_c)(A_{k-1} + W_k) & \text{if the } k^{\text{th}} \text{ maintenance is corrective} \\ (1 - \rho_p)(A_{k-1} + W_k) & \text{if the } k^{\text{th}} \text{ maintenance is preventive} \end{cases}$$

Two approaches to build a GCR model

Based on generalized virtual age models: It is necessary to specify:

- The competing risks model for a new system (IPRA, DT, AD, ...).
- The maintenance efficiencies for each kind of maintenance based on virtual age assumptions.

Based on a reconfiguration of the parameters: It is necessary to specify:

- The competing risks model for a new system: $CR(\Theta_0)$.
- The evolution of the parameters according to the past of the maintenance process (wear-out, efficiency, reactivity, monitoring, ...) $CR(\Theta_k)$.

First classes of GCR models

Conditionally Independent Generalized Competing Risks models (CIGCR - Dijoux, Doyen, Gaudoin [08]): Conditionally to the past $\{\mathbf{W_i}, \mathbf{U_i}\}_{1 \leq i \leq k}$, the risks Y_{k+1} and Z_{k+1} are independent.

$$S_{k+1}(y, z; \mathbf{W}_k, \mathbf{U}_k) = S_{Y_{k+1}}(y; \mathbf{W}_k, \mathbf{U}_k)\ S_{Z_{k+1}}(z; \mathbf{W}_k, \mathbf{U}_k)$$

$\rightarrow$ These models are identifiable.
$\rightarrow$ IUCR+GVA = CIGCR

Generalized Proportional Hazards models (GPH - Deloux, Dijoux, Fouladirad [12]): After the k^{th} maintenance, time to next maintenance W_{k+1} and kind of next maintenance U_{k+1} are independent conditionally to the past.

$$P\left(W_{k+1} > w, U_{k+1} = u | \mathbf{W}_k, \mathbf{U}_k\right) = P\left(W_{k+1} > w | \mathbf{W}_k, \mathbf{U}_k\right) P\left(U_{k+1} = u | \mathbf{W}_k, \mathbf{U}_k\right)$$

$\rightarrow$ PH+GVA=GPH
$\rightarrow$ The maintenance intensities remain proportional.

Generalized Random Sign Models and Generalized Alert Delay Models

Generalized Random Sign models (GRS)

Definition: Conditionally to the past $\{\mathbf{W_i}, \mathbf{U_i}\}_{1\leq i \leq k}$, U_{k+1} and Z_{k+1} are independent.

Properties:

- $\phi_{k+1}(.; \mathbf{W}_k, \mathbf{U}_k)$ is maximum at the origin.
- RS+GVA $\neq$ GRS
- There is a condition of existence of a GRS model similar to the one presented by Cooke [93] for UCR models.

$$\forall t \geq 0, \ \ \forall k \geq 1, \ \ \int_t^{\infty} \lambda^M_{c_k+u}(k; \mathbf{W}_k, \mathbf{U}_k) e^{-\int_t^u \lambda^K_{c_k+v}(k;\mathbf{W}_k,\mathbf{U}_k)dv} du$$

$$< \int_0^{\infty} \lambda^M_{c_k+u}(k; \mathbf{W}_k, \mathbf{U}_k) e^{-\int_0^u \lambda^K_{c_k+v}(k;\mathbf{W}_k,\mathbf{U}_k)dv} du$$

Generalized Repair Alert model

Definition:

- Generalized random sign assumption
- $P(Y_{k+1} \leq y | Z_{k+1} = z, Y_{k+1} < Z_{k+1}, \mathbf{W}_k, \mathbf{U}_k) = \dfrac{G_{k+1}(y; \mathbf{W}_k, \mathbf{U}_k)}{G_{k+1}(z; \mathbf{W}_k, \mathbf{U}_k)}$
  $G_{k+1}(0; \mathbf{W}_k, \mathbf{U}_k) = 0$, $G_{k+1}(.; \mathbf{W}_k, \mathbf{U}_k)$ are increasing.
- $q(\mathbf{W}_k, \mathbf{U}_k) = P(Y_{k+1} < Z_{k+1} | \mathbf{W}_k, \mathbf{U}_k)$

$\rightarrow$ Multiple parametrizations are possible

$\rightarrow$ Possibility to define a Generalized highly correlated censoring model.

Generalized Alert Delay models (GAD)

$$Y_{k+1} = p_{k+1} Z_{k+1} + \mathcal{E}_{k+1}$$

- $\mathcal{E}_{k+1}$ is independent of Z_{k+1} conditionally to the past of the maintenance process
- $p_{k+1} = p(\mathbf{W}_k, \mathbf{U}_k) \in [0, 1]$.

$\rightarrow$ p_{k+1} is related to the PM policy and the monitoring of the system.

$\rightarrow$ The conditional distribution of Z_{k+1} reflects the impact of past maintenances on the risk of failure and the general wear-out of the system.

$\rightarrow$ The conditional distribution of $\mathcal{E}_{k+1}$ reflects the evolution of the reactivity of the maintenance team.

$\rightarrow$ AD+GVA $\neq$ GAD

Examples of GAD models

Exponential GAD models consist in GAD models where the conditional distributions of Z_{k+1} and the conditional distributions of $\mathcal{E}_{k+1}$ are exponential with respective parameters $\lambda_{k+1} = \lambda_{k+1}(\mathbf{W}_k, \mathbf{U}_k)$ and $\mu_{k+1} = \mu_{k+1}(\mathbf{W}_k, \mathbf{U}_k)$.

$\longrightarrow$ Multiple potential parametrizations for λ_{k+1}, μ_{k+1} and p_{k+1} (Dijoux, Gaudoin)

GAD model associated with virtual age

- A new system has a Weibull type hazard rate.
- The effect of CM is of the virtual age type:

$$P(Z_{k+1} > z | \mathbf{W}_k, \mathbf{U}_k) = \frac{S_Z(A_k + z)}{S_Z(A_k)}$$

- The model is GAD : $Y_{k+1} = pZ_{k+1} + \mathcal{E}_{k+1}$, with a constant alert threshhold $p \in [0,1]$.
- The conditional distribution of $\mathcal{E}_{k+1}$ is exponential with parameter μ.

Simulations results and applications to real data

The model

- GAD model $Y_{k+1} = p_{k+1} Z_{k+1} + \mathcal{E}_{k+1}$.
- $p_{k+1} = \delta^{\sum_{i=1}^{k} \prod_{j=i}^{k} (1-U_j)} p$.

  $\longrightarrow$ p is the nominal alert threshold, δ is a parameter related to the impact on the threshold after consecutive failures ($p = 0.8$, $\delta = 0.8$).
- The conditional distribution of Z_{k+1} is exponential with parameter $\lambda_{k+1} = \lambda + \alpha k$.

  $\longrightarrow$ λ is the initial failure rate, α is a parameter related to the ageing of the system ($\lambda = 1$, $\alpha = 0.01$).
- The conditional distribution of Y_{k+1} is exponential with parameter $\mu_{k+1} = \beta^{\sum_{i=1}^{k} \prod_{j=i}^{k} (1-U_j)} \mu$.

  $\longrightarrow$ μ is the nominal delay rate, β is a parameter related to the impact on the delay after consecutive failures ($\mu = 5$, $\beta = 1.2$).

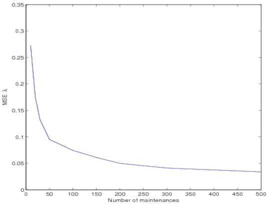

Figure : MSE for $\widehat{\lambda}$

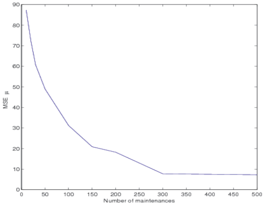

Figure : MSE for $\widehat{\mu}$

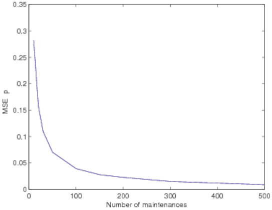

Figure : MSE for $\widehat{p}$

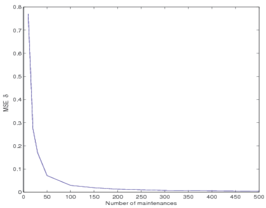

Figure : MSE for $\widehat{\delta}$

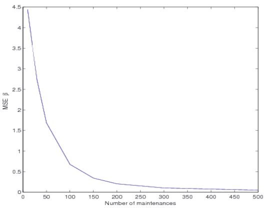

Figure : MSE for $\widehat{\beta}$

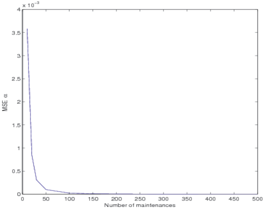

Figure : MSE for $\widehat{\alpha}$

$\longrightarrow$ Rather slow convergence for $\widehat{\lambda}$, $\widehat{\mu}$, rather fast convergence for $\widehat{\delta}$, $\widehat{\alpha}$.

EDF data

→ Dataset provided by EDF.
→ PM and CM times (in days) of a specific component of an electricity production system.
→ 5 PM and 24 CM are observed.
→ The observations are right-censored at time 6113.

CM: 290	CM: 336	CM: 353	CM: 413
PM: 444	CM: 453	CM: 563	CM: 585
...	...	...	...
CM: 6093	Cens: 6113		

Table : EDF dataset

$\widehat{\lambda}$	$\widehat{\mu}$	$\widehat{p}$	$\widehat{\delta}$	$\widehat{\beta}$	$\widehat{\alpha}$
Fail. rate	Delay Rate	threshold	imp. threshold	imp. delay	ageing
0.004	0.0011	0.75	0.13	1.07	$1.7\ 10^{-5}$

Table : Parameter estimation for the EDF dataset

Conclusion and future work

Conclusion

- Development of a wide variety of competing risks models for imperfect maintenance.
- Numerous parametrizations are possible which require expert opinions on the system.
- The parameters allow to describe phenomena not all present for usual competing risks: the intrinsic wear-out of the system, the evolution of the reactivity of the maintenance team, the evolution of the monitoring, PM efficiency and CM efficiency.

Prospects

- Identify a small number of tractable and flexible models, which provide good fit to real data.
- Transcribe faithfully the expert opinions in the modelling (Bayesian approach)
- Build model selection criteria (extend the procedures based on the diagnostic function for UCR models to GCR models)